NEW YORK UNIVERSITY
INSTITUTE OF MATHEMATICAL SCIENCES
25 Waverly Place, New York 3, N. Y.

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Institute of Mathematical Sciences
Division of Electromagnetic Research

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Reflection and Transmission by a Class of Curved Dielectric Layers

S. N. KARP

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REFLECTION AND TRANSMISSION BY A CLASS OF CURVED DIELECTRIC LAYERS

S.N. Karp

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Sidney Borowitz
Acting Director

Dr. Werner Gerbes
Contract Monitor

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Abstract

We consider the reflection and transmission of electromagnetic waves by a special class of curved dielectric layers. The layers in question are bounded by two non-intersecting confocal parabolas, and between these boundaries, the dielectric constant, which may be complex, has a special form of space dependence. It is found that layers of this type have striking properties. If a plane wave is normally incident on such a layer, from the convex side, then the reflected field can be expressed as a product of two factors. The first factor is the field which would be reflected by a conducting parabola coincident with the surface of the layer. The second factor has no space dependence; it depends only upon the spacing of the shell boundaries, the frequency and the special choice of dielectric constant. The transmitted wave has an especially simple form. It is a plane wave. The configuration thus has properties similar to those of a plane dielectric slab. In fact, the plane slab is a limiting case of the geometry we have described.

Table of Contents

	Page
1. Introduction	1
2. Analysis	3
a) Description of the problem	3
b) Mathematical formulation in parabolic coordinates, and solution	5
3. Discussions and conclusions .	11
a) Formulation and interpretation of the principal result	11
b) Comparison with a plane slab	12
c) An open question, and possible application of its resolution	14
d) Extensions, applications and related open questions	15
References	16

1. Introduction

The present paper is concerned with the analysis of the reflection and transmission properties of a special class of curved slabs of material. The dielectric constant is space-dependent in a special way. The purpose of the analysis is to examine the space dependence of the reflected and transmitted waves. We observe that the space dependence of the reflected and transmitted waves due to a plane slab, whose index varies in the direction perpendicular to a face, is very simple and well known; while the dependence on frequency, slab thickness and slab material is elementary only for special cases, such as the case of a constant index of refraction[2]. The principal result of our analysis, which is stated in the abstract, expresses an analogy between the curved slab we treat and plane slab. For the class of parabolic slabs which we consider, the space dependence of the rigorous solution is particularly simple, while the dependence on the frequency, material and thickness is not easily analyzable except in special cases: no such analysis is given here.

A brief description of the slabs we consider and of the results we obtain is given in the abstract which precedes this work. In Section 2 we give a fuller description of the class of slabs to be treated. This class of slabs should be thought of as intermediate in simplicity between the class of plane slabs and that of circular shells or layers of matter. The latter class of geometries presents analytical difficulties* of the type already familiar in the case of diffraction or reflection from perfectly conducting circular cylinders. The parabolic slabs which we consider offer the same

---

*Although the solution can be obtained in the form of an infinite series, special techniques are required for its summation unless the radius is sufficiently small.

sort of simplification as ensues in the theory of diffraction, as a result of the transition from a conducting circular cylinder to a conducting parabolic cylinder. The solution for reflection of an axially incident wave by a conducting parabolic cylinder has been determined in closed form by Lamb [1]. The present analysis involves a procedure similar to that employed by Lamb, and yields a correspondingly simple result.

The plan of our paper is as follows. The description of the problem is to be found in Section 2(a). A reformulation in parabolic coordinates, and the final solution is given in Section 2(b). Section 3 is devoted to a discussion of the results. The results are interpreted in Section 3(a), and compared with the results for a plane slab in Section 3(b). There we also discuss the transition to a plane slab. In Section 3(c) we point out an open question: at a given frequency, can there exist a reflectionless curved slab of the class we consider? This is possible for a plane slab, of course (see [2], p. 513-14). The special usefulness of such a putative reflectionless curved slab is discussed. In Section 3(d) we discuss various extensions and applications of the layer distributions envisioned in the present paper. As we noted in the abstract, when a plane wave is incident normally on a curved slab of the class we described, the transmitted wave is rigorously plane. Since this property may be useful in connection with radome theory and/or practice, a question is raised, but not answered, as to the sensitivity of the effect to small deviations from normal incidence. Still other extensions, applications and unsolved questions, which are mentioned briefly in Section 3(d), will be omitted in this introduction except that we wish to emphasize here that the present results are purely two-dimensional and refer to a special polarization. Although the corresponding scalar problem for a body of revolution can be easily worked on the lines of the present paper, this doesn't seem to be the case for the vector problem.

2. Analysis

a) Description of the problem

We consider the two-dimensional problem of the reflection and transmission of a plane electromagnetic wave by a curved layer of material with a real or complex dielectric constant. The layer (see Figure 1) is confined between two confocal parabolas, whose focus is the origin of a rectangular cartesian xy coordinate system. The axis of symmetry is the x-axis. The respective parabolas are given by the equations

$$x = \frac{y^2}{4\eta_o^2} - \eta_o^2 \quad , \tag{1a}$$

$$x = \frac{y^2}{4\eta_1^2} - \eta_o^2 \quad . \tag{1b}$$

The entire electric field $\vec{E}$ has only a z component $E_z = u(x,y)$. On either side of the layer we have free space. The propagation constant is

$$k = \frac{2\pi}{\lambda} \tag{2}$$

where λ is the wave length, so that outside the layer we have the equation

$$\Delta u + k^2 u = 0 \quad . \tag{3}$$

We assume a time dependence $\exp(-i\omega t)$, with $\omega = kc$, where c is the velocity of light.

Within the layer, the medium differs from free space. The assumption on which our analysis is based consists in postulating that within the layer we have

$$\Delta u + \left[k^2 - \frac{F(\sqrt{r}\,\sin\frac{\theta}{2}, k)}{r}\right] u = 0 \quad . \tag{4}$$

Notice that in (4) the function F is an <u>arbitrary</u> function of its arguments, so that it may be taken to be complex, if desired, and to have any physically admissible frequency dependence. In the above equation, r,θ are the usual polar

coordinates, and

$$x = r\cos\theta; \quad y = r\sin\theta; \quad 0 \leq \theta \leq 2\pi \quad . \tag{5}$$

If the function F is bounded inside the layer then the index of refraction of the layer will tend towards that of free space as the distance from the nose of the layer is increased. Some special realizations of the function F are

$$F = C(k) \ , \quad \text{independent of } r,\theta \ , \tag{6}$$

and

$$F = C(k) \ (\sqrt{r}\ \sin\frac{\theta}{2})^2 \ . \tag{7}$$

Since the index of refraction of the layer corresponding to (4) is $\sqrt{1-(F/k^2r)}$, it is easily seen that , for the special medium assumed in (6) the index will vary along the axis of the layer. On the other hand, the index of refraction corresponding to equation (7), is constant along the axis of the layer, i.e. along the line $\theta = \pi$, $\eta_0^2 < r < \eta_1^2$. It should be noticed that in both cases (6) and (7) the index tends toward unity as we recede to infinity within the range.

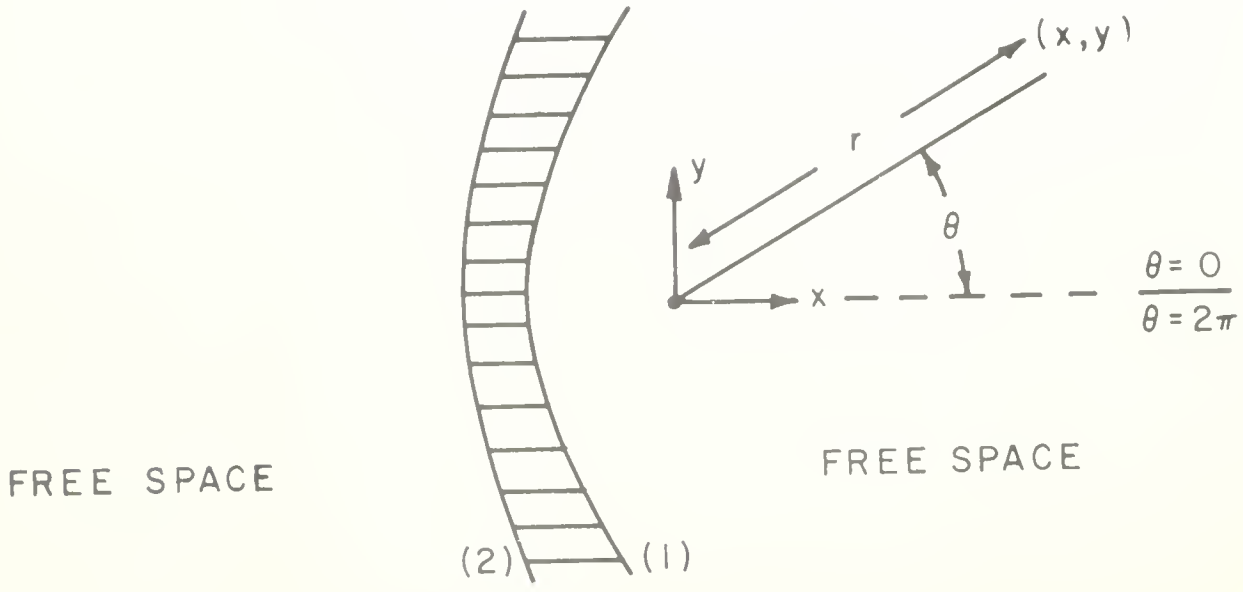

Figure 1.

A plane wave is incident upon the configuration described above. The incident plane wave travels along the positive x-axis and is represented by the equation

$$(8) \qquad E_z^{inc} = u_{inc} = e^{ik(x-ct)} = e^{ikx} \cdot e^{-i\omega t} \ .$$

The time dependence $\exp(-i\omega t)$ will be suppressed in what follows. The incident field is partly reflected and partly transmitted by the layer, and we wish to obtain the total field

$$(9) \qquad u = u^{inc} + (u-u^{inc}) = u^{inc} + u^{resp.}$$

in the presence of the layer. At large distances from the nose the response field is required to be outgoing. In addition, the total field is required to be continuous together with its derivatives, across the parabolas bounding the layer, and regular everywhere else.

b) Mathematical formulation in parabolic coordinates, and solution

In order to treat our problem conveniently, we introduce a substitution employed by Lamb[1], in his discussion of the reflection of a plane wave by a perfectly conducting parabolic cylinder. We introduce a new dependent variable v, and new independent variables ξ and η (parabolic coordinates) in accordance with the equations

$$(10) \qquad u(x,y) = e^{ikx} v(\xi, \eta) \ ,$$

$$\xi = \sqrt{r} \cos\frac{\theta}{2} \ , \quad \eta = \sqrt{r} \sin\frac{\theta}{2} \ ; \ -\infty < \xi < \infty \ ; \ 0 < \eta < \infty \ ,$$

$$x = \xi^2 - \eta^2 \ , \quad y = 2\xi\eta \ .$$

Then it is easily shown that our problem can be expressed in terms of the following equations:

(11) $v_{\xi\xi} + v_{\eta\eta} + 4ik\left[\xi v_{\xi} - \eta v_{\eta}\right] = 0$, for $\eta > \eta_1$, and also for $0 < \eta < \eta_o$,

(12) $v_{\xi\xi} + v_{\eta\eta} + 4ik\left[\xi v_{\xi} - \eta v_{\eta}\right] - p^2(\eta)v = 0$, for $\eta_o < \eta < \eta_1$.

(13) $v\left[\xi_1\ \eta_o-\right] = v(\xi, \eta_o+)$; $v(\xi_1\ \eta_1-) = \bar{v}(\xi, \eta_1+)$,

(14) $\frac{\partial v}{\partial \eta}(\xi, \eta_o-) = \frac{\partial v}{\partial \eta}(\xi, \eta_o+)$; $\frac{\partial v}{\partial \eta}(\xi, \eta_1-) = \frac{\partial v}{\partial \eta}(\xi, \eta_1+)$,

where

(15) $p^2(\eta) = p^2(\eta, k) = 4F(\eta, k)$; $\eta = \sqrt{r}\ \sin(\theta/2)$.

Equation (12) holds inside the layer; equation (11) holds outside the layer, while (13) and (14) express the transition conditions across the boundaries of the layer. In addition we require that the response field, $u-u^{inc}$, be outgoing. Since $u^{inc} = \exp(ikx)$, $u = v \exp(ikx)$ we require that

$$u - u^{inc} = e^{ikx}\left[v-1\right]$$

be outgoing. Also, we have to require the regularity of it and its derivatives except at the boundaries of the layer. It is clear that these requirements can be satisfied by a function v which is independent of ξ. If we introduce this simplification into the equations, they become ordinary differential equations. In fact, (11) is then solvable in terms of Fresnel integrals. Its general solution is

(16) $$v = C + D \int^{\eta^2} \left\{ \left[\exp(2ik\tau)\right] \sqrt{\tau} \right\} d\tau .$$

The requirement that $u = e^{ikx}v$ be a regular function of (x,y) serves to simplify the solution considerably in the range $0 \leq \eta < \eta_o$, since $\frac{\partial \eta}{\partial y}$ is singular at $y = 0$, $0 \leq x \leq \infty$. In fact, since

$$(17) \qquad \frac{\partial u}{\partial y} = e^{ikx}\frac{\partial v}{\partial y} = \left[e^{ikx}\frac{\partial v}{\partial \eta}\right]\frac{\partial \eta}{\partial y} \ ,$$

and since $\frac{\partial \eta}{\partial y}$ is sirgular, the requirement of regularity implies, in view of (16) and (17) that

$$(18) \qquad v = T \qquad \text{for } 0 \le \eta < \eta_o$$

where T is a constant to be determined. The transmitted field therefore has the particularly simple form

$$(19) \qquad u = Te^{ikx} \qquad 0 \le \eta \le \eta_o \ ,$$

and it represents an outgoing wave as is required by physical considerations.

On the other hand, in the region $\eta > \eta_1$, the ξ independent solution of (11) automatically leads to a regular function v(x,y). In this region the decisive requirement is that the field be composed of the given incident wave plus an outgoing wave. The appropriate solution of (11), [cf. equation (16)] turns out to be

$$(20) \qquad v = 1 - \frac{B}{2}\int_{\eta^2}^{\infty} \frac{e^{2ik\tau}}{\sqrt{\tau}}\, d\tau \qquad \eta > \eta_1 \ .$$

Then the complete solution, for $\eta > \eta_1$ has the form

$$(21) \qquad u = e^{ikx} - \frac{B}{2}e^{ikx}\int_{\eta^2}^{\infty} \frac{e^{2ik\tau}}{\sqrt{\tau}}\, d\tau \qquad \text{for } \eta > \eta_1 \ .$$

Here B is a constant to be determined. Notice that at infinity, in the region presently considered we have $\eta > \infty$. Then $u - u^{inc}$, as it should. Furthermore, if we expand the second term of (21) for large , by integration by parts, we find that

$$(22) \qquad u - u^{inc} \to + \frac{B}{4ik} \frac{e^{ikr}}{\sqrt{r}} \csc \frac{\theta}{2}, \quad \text{for} \quad \eta \to \infty \ .$$

Consequently, we see that the response, $u - u^{inc}$, behaves like an outgoing cylindrical wave in the region $\eta > \eta_1$, as we required originally.

We now turn to a discussion of the field inside the layer i.e. in the region $\eta_0 < \eta < \eta_1$. In this region, the field is governed by (12), which, when v is independent of ξ, becomes the following ordinary differential equation:

$$(23) \qquad \frac{d^2v}{d\eta^2} - 4ik\eta \frac{dv}{d\eta} - p^2(\eta)v = 0 , \qquad \eta_0 < \eta < \eta_1$$

where $p^2(\eta) = 4F(\eta,k)$, [cf. equation (15)]. Now let $w(\eta)$ be the solution of (23) which satisfies the initial conditions

$$(24) \qquad w(\eta_0) = 1 , \quad w'(\eta_0) = 0 .$$

Then equations (13), (14), (18) and (24) imply that

$$(25) \qquad v(\eta) = Tw(\eta) \qquad \eta_0 \le \eta \le \eta_1 .$$

To complete our solution we have to apply the transition conditions (13) and (14) at the surface $\eta = \eta_1$, and thereby determine the constants T and B (which occur in (18), (20) and (25). The continuity of v [see (13)] implies that

$$(26) \qquad 1 - \frac{B}{2} \int_{\eta_1^2}^{\infty} \frac{e^{2ik\tau}}{\sqrt{\tau}} d\tau = Tw(\eta_1) ,$$

while the continuity of v_η [see (14)] implies that

$$(27) \qquad Be^{2ik\eta_1^2} = Tw'(\eta_1)$$

(26) and (27) are the equations used to determine the constants B and T. Of course these constants may depend upon k, η_0 and η_1 ; they are constant with respect to the space coordinates.

We shall now show that (26) and (27) are always solvable. We assume at this point that the layer, as expressed by the function $-\frac{F(\eta,k)}{r}$, contains no sources. (Note that the function $\frac{1}{r}$ is infinite only at r = 0, which is outside the layer.) It follows that there can be no outgoing solution of our problem without an incident field. But just such a non-trivial solution could be exhibited in the form

$$v = -\frac{B}{2}\int_{\eta^2}^{\infty} \frac{e^{2ik\tau}}{\sqrt{\tau}}\, d\tau \qquad \eta > \eta_1$$

$$v = T_1\, w(\eta) \qquad \eta_1 > \eta > \eta_0$$

$$v = T_1 \qquad \eta_0 > \eta > 0$$

$$u = e^{ikx} v \quad ,$$

provided that the transition conditions are satisfied, i.e. that we can find non-trivial B_1 and T_1 satisfying the homogeneous linear equation

$$(28) \qquad -\frac{B_1}{2}\int_{\eta_1^2}^{\infty} \frac{e^{2ik\tau}}{\sqrt{\tau}}\, d\tau = T_1 w(\eta_1)$$

$$(29) \qquad B_1\, e^{ik\eta_1^2} = T_1 w'(\eta_1) \quad .$$

Our restriction to a non-radiating layer makes this impossible; hence, the determinant of (28) and (29) is not zero. But these equations have the same determinant as (26) and (27). Therefore (26) and (27) are solvable.

The values of the constants B and T of (26) and (27) are given by the following formulas:

$$B = \frac{w'(\eta_1)}{\left[ww'_o - w'w_o \right] \eta = \eta_1} \tag{30}$$

$$T = \frac{e^{2ik\eta_1^2}}{\left[ww'_o - w'w_o \right] \eta = \eta_1}$$

$$w_o = \frac{1}{2} \int_{\eta_1^2}^{\infty} \frac{e^{2ik\tau}}{\sqrt{\tau}} \, d\tau \ .$$

Of course, the solution as it stands is, necessarily, not explicit since it depends on the choice of $p^2(\eta) = 4F(\eta,k)$, and also on the function $w(\eta)$ which is the solution of the initial value problem

$$w'' - 4ik\eta w' - p^2(\eta)v = 0 \qquad \eta_1 \geq \eta \geq \eta_o \tag{31}$$

$$w(\eta_o) = 1; \ w'(\eta_o) = 0.$$

If we choose $p^2(\eta)$ to be either constant or quadratic in η, the solution may be expressed in terms of confluent hyper-geometric functions, which are, however, not conveniently tabulated for the present purpose. It seems best, at this point, to leave the choice of $p^2(\eta)$ to the radome designer or investigator. But it should be noted that it is possible to generate solvable cases at will by selecting the function $w(\eta)$ a priori and then substituting into (31) and solving for $p^2(\eta)$. Of course, the resulting k dependence must be required to be of a physically realizable kind.

3. Discussion and conclusions

a) Formulation and interpretation of the principal result

Although we do not investigate, in this paper, the detailed form of the constants T and B, we are still able to draw some general conclusions with respect to any layer of the class we have considered. In fact (19) and (20) represent simple formulas for the space dependence of the reflected and transmitted field.

In order to interpret the field in front of the layer [see (21)], note that if we were to remove the layer, and replace it by a perfectly conducting parabolic cylinder at $\eta = \eta_o$, while retaining the same incident wave, the complete solution [1], which we shall call $\tilde{u}$, would be

$$\tilde{u} = \left(1 - \frac{\tilde{B}}{2}\int_{\eta^2}^{\infty} \frac{e^{ik\tau}}{\sqrt{\tau}}\, d\tau\right) e^{ikx} \qquad \eta > \eta_o \tag{32}$$

$$\frac{\tilde{B}}{2} \equiv \frac{1}{\displaystyle\int_{\eta_o^2}^{\infty} \frac{e^{2ik\tau}}{\sqrt{\tau}}\, d\tau} \quad .$$

It is easily verified that $\tilde{u}$, as given in (32), vanishes on the conductor $\eta = \eta_o$. We can therefore introduce

$$u_{refl} = \tilde{u} - u_{inc} = \tilde{u} - e^{ikx} \quad , \tag{33}$$

and then our solution, for $\eta > \eta_1$, as given by (21) can be rewritten as follows:

$$\begin{cases} u = u_{inc} - R\,\tilde{u}_{refl} \\ -R = B/\tilde{B} \end{cases} \qquad \eta > \eta_1 \tag{33}$$

This means that the reflected field for our curved slab is a product of two factors. The first factor, -R, is a function of the frequency, the geometry, and the nature of the material in the slab. This factor has no space dependence. We call the number R the reflection coefficient. The second factor $\tilde{u}_{refl}$ is just the field that would be reflected if the back of the layer were silvered and then the layer itself removed. It is therefore space dependent, but does not depend on the material of the layer.

Let us now examine the transmitted field behind the layer, i.e. the field in the region $\eta_o \geq \eta \geq 0$. This field is given by

$$u = T\, e^{ikx} \qquad \eta_o \geq \eta \geq 0 \,. \tag{19}$$

Of course, it is interpretable as a plane transmitted wave, and it is appropriate to call T the transmission coefficient. Like R, the coefficient T depends solely on the frequencey, the material of the parabolic slab, and also on its nose curvature and thickness. Thus T depends on the function $F(\eta, k)$ and upon k, η_1 and η_o. (See Figure 2 below for a schematic representation of the results of this paragraph.)

b) Comparison with a plane slab

It is instructive to compare the solution we have obtained in this paper with the solution for a plane slab. The behavior of the reflected and transmitted field which we have described above is analogous to the behavior of the fields reflected and transmitted by a straight slab perpendicular to the x-axis, due to the incidence of the wave $E_z = e^{ikx} = u^{inc}$. The plane slab may be allowed to have an index of refraction which depends upon x, and upon frequency, and which may be complex. For both the plane and curved slabs, the transmitted field is given by the incident field multiplied by a transmission coefficient. In both cases the reflected field is the

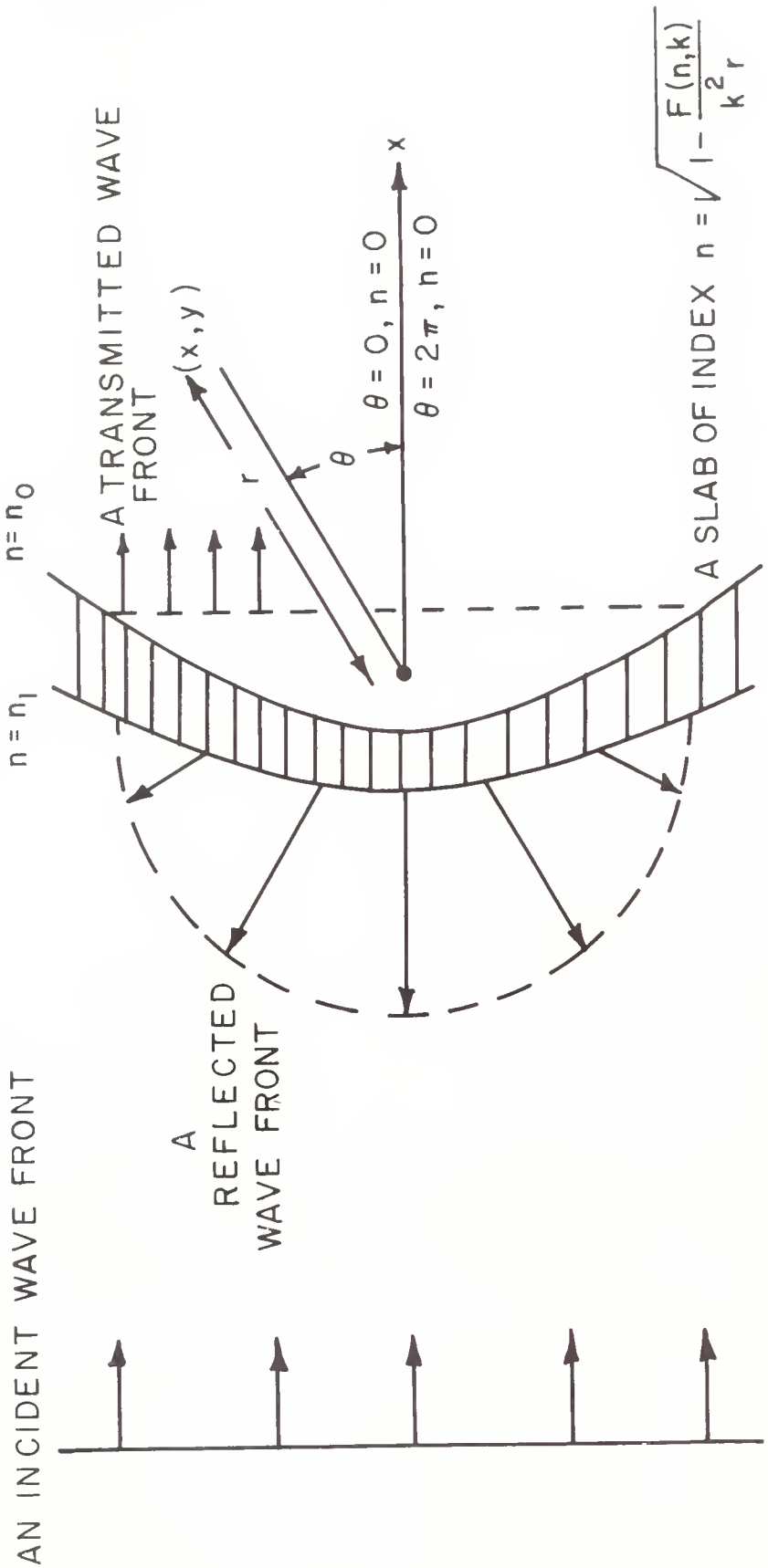
AN INCIDENT WAVE FRONT
A REFLECTED WAVE FRONT
n = n1
n = n0
A TRANSMITTED WAVE FRONT
(x, y)
r
θ
θ = 0, n = 0
θ = 2π, n = 0
x
A SLAB OF INDEX n = √(1 − F(n,k)/k²r)

Figure 2.

product of a space independent but slab dependent reflection coefficient and the field which would be reflected if the back surface of the slab were silvered and then the slab removed. Of course, the space dependent factor of the reflected field for a slab is itself a plane wave, while for the parabolic slab the space dependent factor is a cylindrical wave. Inside either slab the field is of the form

$$u = e^{ikx}v$$

where v obeys a certain second order differential equation. The equations inside the slab for plane and curved slabs respectively, are quite similiar in form. In fact, if we replace η by a constant in (31) we get an equation which has the same form as that which occurs for a plane slab. It is interesting to note that the curved slab becomes more and more plane as we increase η_o and η_1 while maintaining a finite nose thickness d. Here $d = \eta_1^2 - \eta_1^2$. Thus the plane slab is a limiting case of the parabolic slab. It follows that the reflection coefficient for the curved slab ought to approach the reflection coefficient for the corresponding plane slab if we let $\eta_1^2 - \infty$, $\eta_o^2 - \infty$, and $\eta_1^2 - \eta_o^2 = d$. The present writer has verified this limiting behavior in the case of the choice of index corresponding to either

$$a) \quad F(\eta) = \frac{p^2(\eta)}{4} = (d+\eta_o^2)(k^2-k_1^2)$$

$$b) \quad F(\eta) = \frac{p^2(\eta)}{4} = (\eta^2)(k^2-k_1^2) .$$

Here k_1 is a constant, and a) and b) correspond to the functions given in (6) and (7) above. In case b) the index is exactly $\frac{k_1}{k}$ along the axis of symmetry. The procedure employed for studying the limiting case will now be outlined.

First notice that if η_o is large, the original equation (4) approximates to

$$u_{xx} + u_{yy} + \left[k^2-(k^2-k_1^2)\right]u = u_{xx} + u_{yy} + k_1^2 u = 0$$

in the interior of the curved slab, near the nose. Thus, the curved slab approximates to a straight slab of thickness d and of index (k_1/k), lying between the planes $x = -\eta_1^2$ and $x = -\eta_o^2$. If we introduce the new variable

$$t = \frac{\eta^2 - \eta_o^2}{d} \quad , \qquad \eta_o \geq \eta \geq \eta_o \quad , \quad 1 \geq t \geq 0 \quad ,$$

where

$$d \equiv \eta_1^2 - \eta_o^2 \quad ,$$

in (31) we can solve this equation by perturbation theory, using an expansion in inverse powers of η_o^2. The limiting result is that the reflection coefficient is found to be just the reflection coefficient for a plane slab of index $\left(\frac{k_1}{k}\right)$ and thickness d. Higher order approximations can be carried out, but this is outside the scope of the present paper. Here, we must content ourselves with noting the possibility of obtaining, by this means, approximate results which would express the reflection coefficients for the curved slab of our type in terms of its plane counterpart.

c) An open question, and possible application of its resolution

It would be interesting to know the extent to which we can make R small or zero, by proper choice of η_o, η_1, k and $F(\eta,k)$. (This is known to be possible for a plane slab.) The reason for our question is the following. We have already seen that our curved slab always transmits a pure plane wave, but reflects a cylindrical wave. If we could

make $R = 0$, then we could reverse the process, so that a plane wave e^{-ikx} originating in the interior of the parabola would emerge as a plane wave. The properties already demonstrated would seem to be useful in connection with radomes. Hence the usefulness of the layer would be greatly enhanced if we could ensure $R = 0$, i.e., undistorted emergence.

d) Extensions, applications and related open questions

The analysis we have given above easily extends to cover the case of several parabolic slabs. It can be shown that the results obtained in Section 2 as to the nature of the reflected and transmitted wave remain unchanged. Another modification is to make the index behind the layer different from free space. In particular, we can consider the case of an infinitely conducting inner region. This means a perfect conductor at $\eta = \eta_0$. The problem can then be solved in the same manner, and the reflected field is of the same nature. This analysis has been carried out by the author in connection with the analysis of the effect, upon back scattering, of a layer of ionized matter contained between a shock front and a conductor.

Another application of the present theory is to the theoretical assessment of radomes. The advantage of the configuration here treated is that, unlike the solution for the case of two concentric circularly cylindrical boundaries, the solution is simple to handle, and refers to a geometry in which there is no back reflection from an interior source. Application of the reciprocity theorem leads to the result that the transmitted far field amplitude in the direction $\theta = \pi$, due to a line source anywhere inside the radome ($\eta_0 \geq \eta \geq 0$), is constant in absolute value and equal to $|T|$. This result may be applicable to radomes of character other than the one considered here, provided the source is on the axis of symmetry, and the frequency high enough. The further experimental and theoretical analysis of this geometry may be of more than academic interest.

The reader is invited to note that the results given here have been two dimensional, and are restricted to a particular polarization. The general three dimensional vector problem, for a slab or for a shell in the form of a paraboloid of revolution has not been treated here. In addition, notice that there is an open question as to the stability of the results we have obtained. By stability we mean sensitivity to changes of angle of incidence. Doubtless the transmitted wave is no longer a plane wave in our configuration, if the angle of incidence is varied even slightly. But it may be that, by proper choice of index of refraction one can ensure that the deviation from a plane wave can be made small, or even to vanish to first order.

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